TONBRIDGE SCHOOL

Scholarship Examination 2006

MATHEMATICS I

Tuesday 2nd May 2006
9.00 am

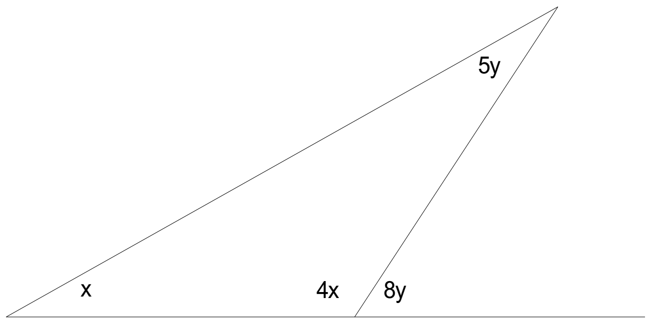
Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. The diagram below shows four angles associated with a triangle. Form two simultaneous equations and solve them to find the values of x and y.

[8 marks]



2. Consider the following sequence of sums.

Sum 1: $\frac{1}{1\times 3} = \frac{1}{3}$

Sum 2: $\frac{1}{1\times 3} + \frac{1}{3\times 5} = \frac{6}{15} = \frac{2}{5}$

Sum 3: $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} = \ldots = \ldots$.

(a) Work out, and simplify, the answer to Sum 3.

(b) Work out, and simplify, the answer to Sum 4.

(c) Use the pattern in your answers to predict the answer to Sum 50.

(d) If the answer to Sum n has denominator 257 when simplified, what is the value of n?

[8]

3. In appropriate units, the height, H, of water waves on a long straight canal of width W and depth D is given by the formula $H^2 = \frac{25}{W\sqrt{D}}$.

(a) If $W = 4.2$ and $D = 9.5$, find H.

(b) If $H = 2.3$ and $D = 5.1$, find W.

(c) If $H = 2.9$ and $W = 3.6$, find D. [8]

4. A wet sponge has a mass of 650 grams; 99% of this mass is water. Some water is squeezed out of the sponge after which only 95% of the mass of the wet sponge is water. How many grams of water were squeezed out of the sponge?

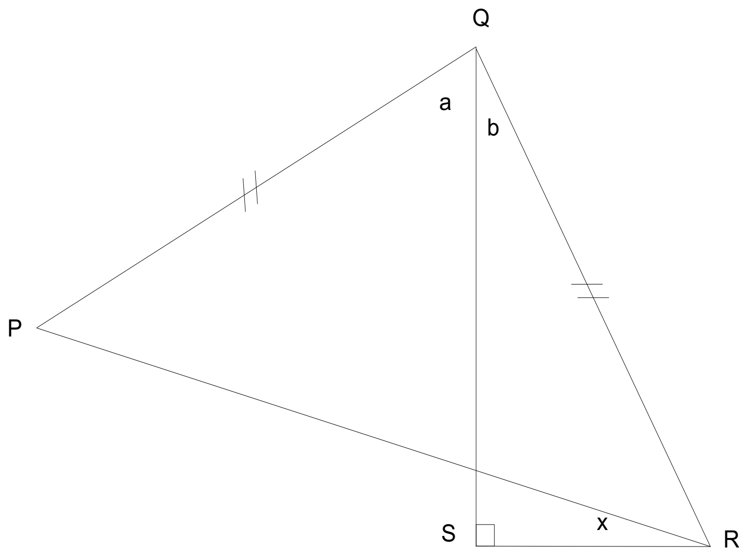
[8]

5. In the diagram below, PQR is an isosceles triangle and QRS is a right-angled triangle: x is angle PRS.

(a) If $a = 65°$ and $b = 45°$, find x.

(b) In general, find the value of x in terms of a and b.

[8]



6. Two right-angled triangles, ABD and BCD, are joined together as shown in the diagram below. AD has length 7 cm, DC has length 17 cm; AB and BC have the same length, x cm.

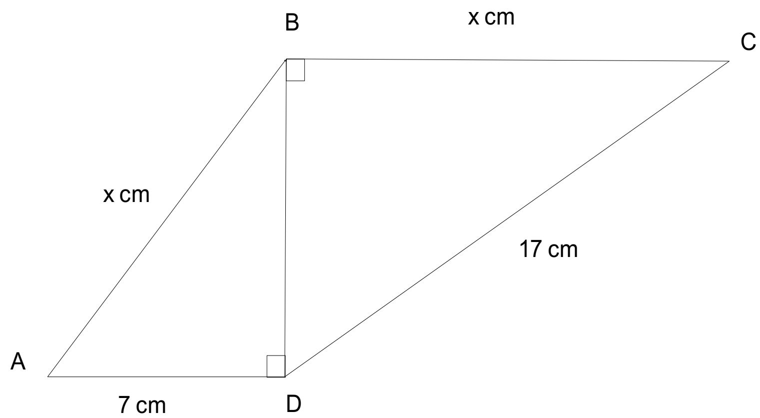
(a) Use Pythagoras' theorem in triangle ABD to find an expression for BD^2.

(b) In a similar way, use triangle BCD to find another expression for BD^2.

(c) Use your answers to (a) and (b) to find the value of x.

(d) Find the length of AC. (Do this by calculation, not by scale drawing.)

[15]



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7. A solid cuboid has a square base of side-length x cm and total surface area 72 cm^2.

(a) Why is the volume equal to 0 cm^3 when (i) $x = 0$, (ii) $x = 6$?

You are given that the volume, y cm^3, of the cuboid is given by the formula $y = \frac{1}{2}x(36 - x^2)$.

(b) When $x = 4$, show that $y = 40$.

(c) Find the values of y corresponding to $x = 0,\ 1, 2, 3, 3.5, 4, 4.5, 5, 6$.

(d) Choosing sensible scales, plot a graph of y against x.

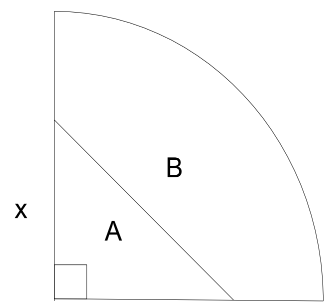
(e) Which value of x gives the maximum volume of the cuboid?

(f) What is special about the cuboid in (e)? [15]

8. The diagram below shows a quadrant of a circle of radius 10 cm together with a right-angled isosceles triangle of side-length x cm which divides the quadrant into two pieces labelled A and B.

(a) Find x if A and B have the same area.

(b) Find x if A and B have the same perimeter.



[15]

9. In this question, we shall call a whole number a *diff-square number* if it can be written as the difference of two square numbers. For example, 4 and 8 are both diff-square numbers because $4 = 2^2 - 0^2$ and $8 = 3^2 - 1^2$.

(a) Make a list of all the square numbers from 1^2 up to 11^2 and use this to decide which of the whole numbers from 1 up to 20 are diff-square numbers.

(b) Consider those numbers from 1 up to 20 that are *not* diff-square numbers. Explain carefully what these numbers have in common.

(c) Use your answer to (b) to predict which of the following are diff-square numbers: (i) 123, (ii) 1234, (iii) 12345, (iv) 123456.

[15]